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OPTIMAL STRATEGIES AND OPTIMAL ROUTES IN PUBLIC TRANSIT ASSIGNMENT MODELS: AN EMPIRICAL COMPARISON

by

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ABSTRACT

We present a brief review of traffic assignment models for transit networks. Then we focus our attention on two transit route choice models: one that is based on the concept of optimal strategy and the other based on the concept of routes perceived to be minimal. In both models, the transit riders are assumed to select a set of attractive transit lines for their trip from origin to destination; the models differ in the way that this subset of lines is selected. The mathematical formulations, and the corresponding solution algorithms are presented. Then we present the transit flows obtained with these algorithms when applied to the public transit system of Santiago de Chile and compare the differences of the simulated flows.

Key words : transit assignment; linear programming.

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RÉSUMÉ

Nous présentons un bref aperçu des modèles d'affectation pour les réseaux de transport en commun. Ensuite, nous présentons en détail deux modèles de choix de route sur ces réseaux : un basé sur le concept de stratégie optimale et l'autre basé sur le concept de route optimale. Dans les deux modèles les voyageurs choisissent un sous-ensemble de lignes de transport en commun attrayantes pour leur déplacement de l'origine à la destination; les modèles diffèrent dans la définition du sous-ensemble de lignes choisi. Les formulations mathématiques des deux modèles ainsi que les algorithmes de résolution sont présentés. Enfin, nous présentons les résultats obtenus avec les deux algorithmes pour le réseau de transport en commun de la ville de Santiago, Chili, et nous comparons les flots simulés.

Mots-clés : affectation du transport en commun; programmation linéaire.

1. INTRODUCTION

The route choice (or traffic assignment) on a public transit network has been studied by several authors. Even though several transit assignment algorithms have been proposed, only in a few cases has a mathematical model been formulated.

The main differences among the assignment algorithms are the hypotheses made, explicitly or implicitly, on the users' behaviour when faced with a route choice. The simplest and most naive approach is to assume that all passengers travelling from origin A to destination B choose one itinerary that minimizes the total travel cost. In this "all or nothing" approach alternative itineraries are not considered.

Le Clercq (1972) was the first to introduce the concept of common lines, that is, lines with different frequencies and travel times that users may include in their route choice. Le Clercq's algorithm considers that all lines that share the same pair of nodes are used by the transit riders on that segment, whereas Chriqui (1974) proposed that only a subset of these lines minimize the total travel cost. The algorithm of Chriqui and Robillard (1975), which implements Chriqui's hypothesis, solves efficiently a hyperbolic programming problem at each node, where common lines occur, in order to determine for all O-D pairs the optimal subsets of common lines in the itineraries used. Both Chriqui's and Chriqui and Robillard's algorithms suppose that, at each stop, travellers board the first vehicle to arrive from the optimal set of common lines. The demand is then assigned proportionally to the frequencies of the lines in the optimal set.

Spiess (1983) introduced the notion of strategy, that may be considered as a generalization of Le Clercq's and Chriqui's concept of route. Based on the assumption that passengers only have the information

of which bus line arrives next at a given stop, Spiess defines a strategy as a set of rules that allow the passenger to travel from the origin to the destination. The transit assignment problem that determines the optimal strategies is formulated as a linear programming problem. The solution algorithm of this model is included in the commercial code EMME/2. Spiess also proposed a non linear extension for the cases where the average travel costs depend on the number of passengers transported on a line segment, similar to the network equilibrium model for road networks.

Later, Nguyen and Pallotino (1985), characterized a strategy, by using graph theoretical concepts, to be a hyperpath. A hyperpath contains all arcs of a strategy. They showed that if the structures of flow and cost are defined properly, traffic equilibrium models for public transportation networks may be formulated. In those models, the vector of flows on the network is replaced by a vector of flows on the hyperpath. We must note here that Nguyen and Pallotino's model is equivalent to Spiess' model although the solution algorithms proposed are different. Finally, De Cea (1986) formulated a linear programming model of transit assignment which is the model solved by Chriqui's solution algorithm. This formulation was possible by a particular definition of a public transportation network, and is inspired by the approach taken by Spiess.

The paper is organized as follows: in section 2, we define and illustrate the concepts of perceived minimal route and optimal strategy. Section 3 is devoted to a brief presentation of both mathematical models and the main characteristics of their solution algorithms. Section 4 gives a brief description of the solution algorithms of both models. In section 5, we present a case study where both algorithms were applied to the public transit network of the city of Santiago de Chile. Finally, the conclusions of their work are presented in Section 6.

2. THE CONCEPT OF ITINERARY, ROUTE, AND STRATEGY

In the analysis of public transit assignment models it is assumed that passengers follow either itineraries, routes, or strategies of minimal cost (time). We establish clearly their meaning by using a small example transit network.

Figure 1 represents a very simple network composed of 6 lines and 6 nodes. Link travel times are in minutes and we assume an equal frequency of 5 veh/hr. for all lines. In order to compute total travel time, waiting time is assumed to be half the headway of the line, i.e., 6 minutes.

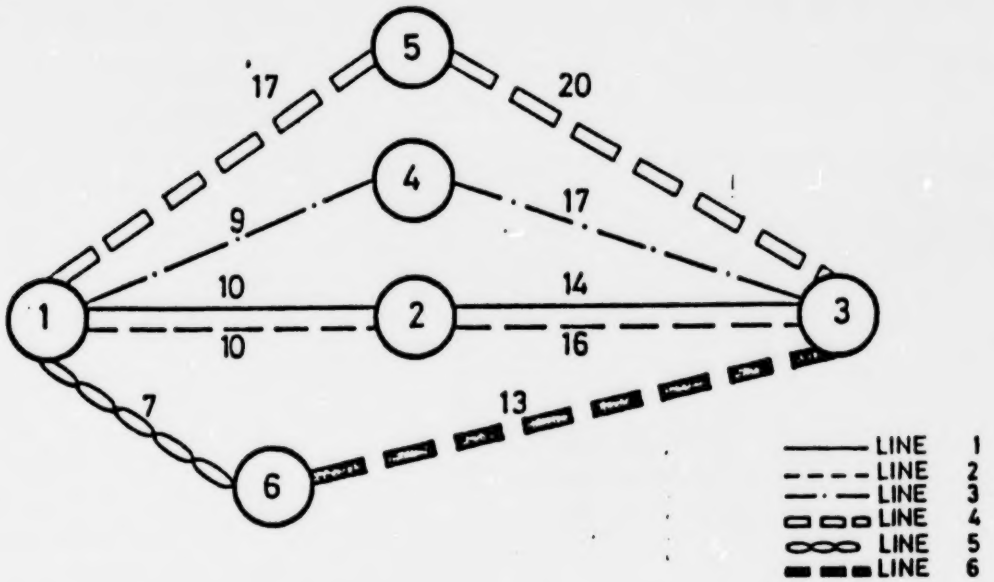


Figure 1: Illustration of Itinerary, Route, and Strategy

Itinerary: detailed description of a trip, where boarding lines and boarding and alighting nodes are specified. In figure 1, the following are itineraries for a trip from 1 to 3: a) line 1 from 1 to 3; b) line 5 from 1 to 6 and line 6 from 6 to 3.

Route : a sequence of nodes given by the origin of the trip, the transfer nodes (if any), and the destination nodes. A part of a route between two consecutive nodes is called a route section. With each node of the sequence a set of attractive lines is associated which allows the passenger to reach the next node in the sequence. In our example, a route going from 1 to 3 is defined, for instance, by 1(L1, L2, L3, L4), 3 or by 1(L5), 6(L6), 3. Passengers are assumed to board the first vehicle to arrive among the set of attractive lines at each boarding node of the trip.

Strategy : set of rules allowing a passenger to travel from the origin to the destination. In figure 1, a possible strategy is the following: at node 1 take the first arriving vehicle. If it is line 1, 2, 3, or 4 alight at node 3. Otherwise, transfer to line 6 at node 6 and then alight at node 3.

From our example it is easy to determine the minimum strategy, route, and itinerary. Table 1 contains the results of an assignment for each case.

Table 1

	description line	from	to	time (min.)	assignment %	line	from	to
minimal itinerary	1	1	3	30	100	1	1	3
perceived minimal route	1,2,3	1	3	27 $\frac{1}{3}$	33 $\frac{1}{3}$	1	1	3
					33 $\frac{1}{3}$	2	1	3
					33 $\frac{1}{3}$	3	1	3
minimum strategy	1,2,3	1	3	27	25	1	1	3
					25	2	1	3
	5	1	6		25	3	1	3
	6	6	3		25	5	1	6
					25	6	6	3

The results of this example clearly show the differences expected on total travel time and assigned volumes when using itineraries, routes and strategies.

3. MATHEMATICAL FORMULATION OF THE MODELS

3.1 Transit network representation

Before stating the mathematical formulation of the transit assignment models for optimal strategies and minimum routes, a clear definition of the transit network for each case is needed. In addition, line segments and line sections, inherent to these formulations, must be defined as well.

Spiess considers a transit network defined by a graph G=(N,A) where N represents the set of nodes of the network and A the set of links or line segments. A line segment is a portion of any transit line between two consecutive nodes; each link has a travel time and a frequency.

The formulation of the Chriqui's model, given in De Cea (1986), is based on a rather different definition of the transit network. A graph G=(N,L) is defined, where N represents, again, the set of nodes, while L is the set of all possible line sections over the network, a line section being any portion of a transit line between two nodes, which need not be adjacent.

Figure 2a shows a very simple network with four nodes and four transit lines, coded in terms of nodes and line segments. Figure 2b represents the same network coded in terms of nodes and line sections.

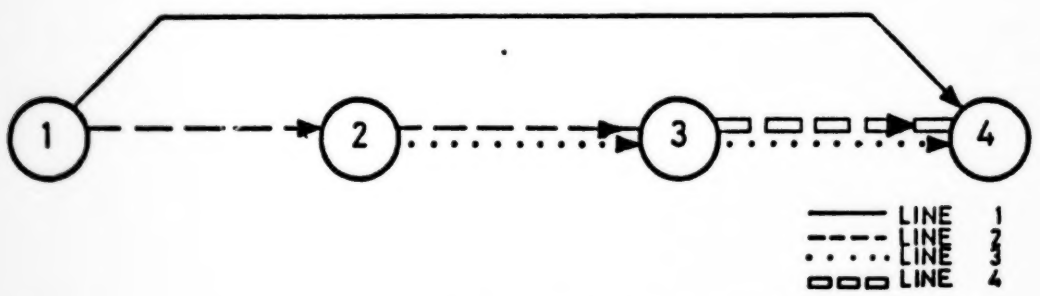


Figure 2a: Graph Based on Line Segments

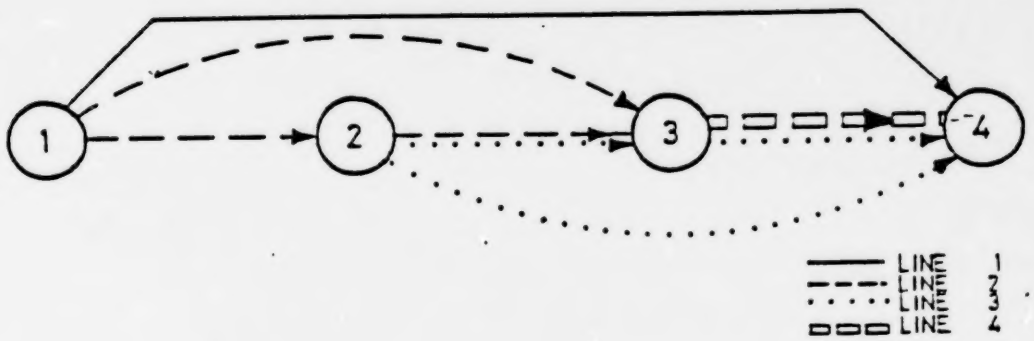


Figure 2b: Graph Based on Line Sections

3.2 Mathematical Formulation of the Spiess' model

a) Notation

Spiess (1983) showed that the transit route choice problem may be stated as a linear programming problem. For the developments necessary to arrive at this relatively simple formulation, see Spiess (1983).

The notation used in this case is the following:

G=(N,A): graph with N representing the set of nodes and A the set of links or line segments.

A_i^+ : set of outgoing arcs from node i.

A_i^- : set of incoming arcs to node i.

t_a : in vehicle travel time on arc a∈A; an arc is a line segment of some line.

v_a : flow on arc a.

f_a : frequency associated to the arc a.

g_i : number of trips from node i to destination d.

v_i : flow on node i.

The model statement is then

$$\text{Minimize} \quad \sum_{a} t_a v_a + \sum_{i \in N} w_i \tag{1}$$

P(1)

$$\text{subject to} \quad \sum_{a \in A_i^+} v_a - \sum_{a \in A_i^-} v_a = g_i \quad , i \in N \tag{2}$$

$$v_a \leq f_a w_i \quad , a \in A_i^+, \; i \in N \tag{3}$$

$$v_a \geq 0 \quad , a \in A \tag{4}$$

The first term of the objective function represents the in-vehicle travel time and the second the expected waiting time, at boarding nodes.

The dual problem of P(1) is:

$$\text{Maximize} \quad \sum_{i \in N} g_i \mu_i \tag{5}$$

$$\text{subject to} \quad \mu_i - \mu_j - m_a \leq t_a \quad , a \in A \tag{6}$$

P(4)

$$\sum_{a \in A_i^+} f_a m_a = 1 \quad , i \in N \tag{7}$$

$$\mu_a \geq 0 \quad , a \in A \tag{8}$$

where the dual variables μ_i, μ_j are associated to nodes and m_a to transit line segments. If v^*, w^*, and μ^* are optimal solutions of P(3) and P(4), then complementary slackness conditions are:

$$(v_a^* - f_a w_i^*) m_a^* = 0 \quad , a \in A_i^+, \; i \in I \tag{9}$$

$$(\mu_j^* + t_a + m_a^* - \mu_i^*) v_a^* = 0 \quad , a \in A \tag{10}$$

The interpretation of the dual variables is rather straightforward: μ_i, μ_j represent the expected total travel time from node i(j) to the destination and m_a is the expected waiting time on link (line segment) a, conditional on the line of link a to arrive first.

3.3 Mathematical Formulation of Chriqui's Algorithm

The formulation of the transit route choice model for the behavioural hypothesis stated by Chriqui (1974) given by De Cea (1986) is based on a different definition of the transit network. The network G=(N,L) consists of the nodes N and line sections L where a line section is defined to be a sequence of transit line segments that a traveller may use without effectuating a transfer. The set L is much larger than the set A in Spiess' model as can be seen from the example in Figure 2.

The model suggested by De Cea uses the following notation:

$G=(N,L)$: graph with N representing the set of nodes and L the set of line sections;

S : set of origin-destination pairs connected by, at least, one line section;

S_{ij} : set of line sections connecting directly nodes i and j;

L_i^+ : set of outgoing line sections from node i;

L_i^- : set of incoming line sections to node i;

v_ℓ : flow on the line section $\ell \in L$;

t_ℓ : in vehicle travel time on the line section $\ell \in L$;

f_ℓ : frequency associated to line section $\ell \in L$;

g_i : number of trips from the origin to destination node i;

v_{ij} : total flow on route section (i,j);

w_{ij} : the expected waiting time on route section (i,j).

The model suggested by De Cea is the following:

$$\text{Min.} \quad \sum_{l \in L} v_l \, t_l + \sum_{(i,j) \in S} w_{ij} \tag{11}$$

$$\text{s.t.} \quad \sum_{l \in L_i^+} v_l + g_i = \sum_{l \in L_i^-} v_l \,, \quad i \in N \tag{12}$$

P(3)

$$v_l \leq f_l \, w_{ij} \,, \quad (i,j) \in S, \; l \in S_{i,j} \tag{13}$$

$$v_l \geq 0 \,, \quad l \in L \tag{14}$$

The dual problem of P(1) is the following:

$$\text{Max.} \quad \sum_i g_i \, \mu_i \tag{15}$$

$$\text{s.t.} \quad \mu_i - \mu_j - m_l \leq t_l \,, \quad l \in L \tag{16}$$

P(4)

$$\sum_{l \in S_{ij}} m_l \, f_l = 1 \,, \quad (i,j) \in S \tag{17}$$

$$m_l \geq 0 \,, \quad l \in L \tag{18}$$

Let v^*, w^*, μ^* and m^* be the optimal solutions of P(3) and P(4). Then, the complementary slackness conditions are

$$(v_l^* - f_l \, w_{ij}^*) \, m_l^* = 0 \,, \quad l \in S_{ij}, \; (i,j) \in S \tag{19}$$

and

$$(\mu_j^* - \mu_i^* + m_l^* + t_l) \, v_l^* = 0, \quad l \in L \tag{20}$$

The interpretation of the dual variables is as in model P(2).

3.4 Comparison of the Models

The mathematical formulations of the strategy and route choice models presented in sections 3.2 and 3.3 are quite similar. In both cases, the dual variables have a similar meaning and, in fact, one may consider the route choice model to be the strategy choice model formulated on a different network.

However, there are important differences on the size of each model, the complexity of the algorithms proposed to solve them and on the solutions obtained. The number of variables and constraints is respectively

$$\|A\|+\|N\|-1 \text{ and } 2\|A\|+\|N\|-1$$

for Spiess' model (P1) and

$$\|L\|+\|S\|-1 \text{ and } 2\|L\|+\|N\|-1$$

for the De Cea's model (P3).

As the cardinality of the line sections set, $\|L\|$, is much greater than the cardinality of the line segments, $\|A\|$, the size of model (P3) is greater than the size of model (P1).

On the other hand, as shown in section 4, the solution algorithm of Chriqui and Robillard must solve explicitly the problem of obtaining the optimal sets of common lines for each set S_{ij}. The algorithm proposed by Spiess does not need to do so and the optimal strategies are calculated by a backward labeling procedure. Thus, Spiess' algorithm may be expected to require less computation time.

As we mentioned above, the concept of strategy may be viewed as a generalization of the concept of route. This means that in theory, the solutions obtained with Spiess' algorithm should exhibit a greater dispersion of trips among lines that are not common to specific lines, than the solution produced by Chriqui and Robillard's algorithm.

For a given network, the magnitude of this difference depends on the specific characteristics of the problem. For example, in situations where the fares are independent of the number of transfers and the number of trips that use more than one line are frequent, the differences between the assignments obtained by each model are likely to be

significant. However, if there is a low proportion of trips with transfers and the users have to pay each time they board a vehicle, the results of both models are likely to be very similar. This remark will be confirmed in section 5 where we present the results obtained by both algorithms, when applied to a network model of the Santiago Public Transport System.

4. SOLUTION ALGORITHMS

4.1 Spiess' algorithm

We present now the algorithm developed by Spiess to solve the transit assignment problem (1)-(4). The algorithm consists of two parts. In a first pass, from the destination node to all origins, the optimal strategy $\bar{A}^*$ and the expected travel and waiting times u_i^* are computed for each node $i \in I$ to the destination r; in a second pass, from all origins to destinations, the demand is assigned to the network according to the optimal strategy.

The detailed steps of the algorithm are as follows:

Part 1: Find optimal strategy

1.1 (Initialization) $u_i := \infty$, $i \in N-\{r\}$; $u_r := 0$

$f_i := 0$, $i \in N$

1.2 (Get next link) If $s = \phi$ STOP

else find $a=(i,j)$ which satisfies

$u_j + t_a < u_{j'} + t_{a'}$, $a'=(i',j') \in S$,

$S := S-\{a\}$.

1.3 (Update node label) If $\mu_i > \mu_j + t_a$ then

$$\mu_i := \frac{f_i\mu_i + f_a(\mu_j + t_a)}{f_i + f_a}$$

$$f_i := f_i + f_a;\ \bar{A} := \bar{A} + \{a\}$$

go to step 1.2.

Part 2: Assign demand according to optimal strategy

2.1 (Initialization) $v_i := g_i$, $i \in N$

2.2 (Loading) Do for every link $a \in A$ in decreasing order of $(\mu_j + t_a)$:

if $a \in \bar{A}$, then $v_a := \frac{f_a}{f_i} V_i$,

$$V_j := V_j + v_a$$

else $v_a := 0$.

In step 1.1, the node labels μ_i, i.e. the expected travel times to reach the destination, are set to infinity for all nodes except the destination, for which it is initialized to zero. The auxiliary variables f_i, $i \in N$, that contain the combined frequencies of lines selected at node i, are initialized to zero. The set S is used to identify the links that have not yet been examined and the set $\bar{A}$ is used to identify the optimal strategy.

In step 1.2, the link nearest to the destination is selected among the links not yet examined. The time considered is $\mu_j + t_a$, that is, the time from the i-node of the link to the destination not including the waiting time at node i. If this is smaller than the current time associated with node i, μ_i, link a is included in the optimal strategy and μ_i and f_i are updated according to the formulae given in step 1.3.

Note that where the label of a node i, μ_i, is improved for the first time, $f_i \mu_i = 0 \cdot \infty$. In order to keep the algorithm as compact as possible, the convention $0 \cdot \infty = 1$, whenever it occurs. Part 1 of the algorithm terminates when all links have been examined.

In the second part of the algorithm the demand from node i to the destination, g_i, is assigned to the network according to the strategy $\bar{A}$. This is done by assigning to each link $a \in \bar{A}$ the proportion of the node volume that corresponds to its frequency. Since the links are processed in reverse topological order (decreasing $\mu_j + c_a$), the node volumes may be updated in parallel, hence making it possible to examine every link only once.

The complete proof of optimality of the solution computed by the algorithm is rather complex and may be found in Spiess (1983).

4.2 Chriqui's algorithm

The route generating for shortest path algorithm proposed by Chriqui is an adaptation of Dijkstra's algorithm. The labeling of a node involves the solution of an unrestricted (0,1) hyperbolic programming problem that selects the set of attractive lines at a node (see Chriqui, 1974). Even though this is done in a very efficient way, the labeling process is very time consuming when applied to large scale transit networks that are characterized by the presence of numerous common lines at nodes.

The label associated with each node j of the network is permanent or temporary whether or not node j has become part of the set of minimal time routes. The label attributes are the following:

μ_j: total travel time from the origin θ (tree root) to node j;

k_j: last transfer node in the route from θ to j;

L_j: set of line sections ($L_j \subset L$) that minimize the total travel time from k_j to j. This set, that we denote $S^*_{k_j j}$, is a subset of $S_{k_j j}$ which contains all the line sections connecting k_j and j, sorted in ascending order of their vehicle travel times. In order to make the presentation of the algorithm clear, we suppose that all the sets S_{ij} are known (usually the sets S_{ij} are identified by a simple search of the transit network data structures).

The algorithm is presented below by using the notation defined in section 3.3 and the following auxiliary variables:

TV_{pj}: total travel time on the route section from p to j;

f_{pj}: combined frequency of the route section from p to j which corresponds to the sum of the frequencies of all the line sections belonging to S^*_{pj}.

Let S^*_{pj}, f_{pj}, and TV_{pj} be the solution of $[S_{pj}, S^*_{pj}, f_{pj}, TV_{pj}]$, the hyperbolic problem which gives the set of line sections that minimize the total travel time, TV_{pj}, between nodes p and j.

Chriqui's algorithm can then be written as follows:

Part 1: Find optimal routes

1.1 (Initialization) $\quad \mu_j := \infty,\ j \in N-\{\theta\}$;

$\mu_\theta := 0$; $k_\theta := \theta$; $L_\theta = f$;

$X := \emptyset$

1.2 (Determination of a new pivot node)

If $\bar{X} := \emptyset$ STOP

else find node p which satisfies

$\mu_p := \min\{\mu_i, i \in \bar{X}\}$.

1.3 (Labeling nodes directly reachable from p)

For every node j, directly reachable from p

compute $[S^*_{pj}, f_{pj}, TV_{pj}]$; then

if $\mu_p + TV_{pj} < \mu_j$ then

$\mu_j := \mu_p + TV_{pj}$, $k_j := p$; $L_j = S^*_{pj}$

else continue

$X = X \cup \{p\}$

go to step 1.2.

Part 2: Assign demand according to optimal routes

2.1 (Initialization)

$v_\ell := 0$, $\ell \in L$

2.2 (Loading)

For every destination node j

2.2.1 (Demand for route θ,j)

$g := g_j$

2.2.2 (Load route sections)

$d := j$; $e := k_d$; $v_\ell = v_\ell + \frac{f_\ell}{f_{ed}} \cdot g$, $\ell \in S^*_{ed}$

$j := e$

if $j \neq 0$ then go to 2.2.2

else continue.

The algorithm that solves the hyperbolic problem $[S_{pj}, S^*_{pj}, f_{pj}, TV_{pj}]$ is as follows:

1.0 (Initialization)

$S^*_{pj} := \emptyset$; $f_{pj} := 0$;

2.0 (Compute total travel time for the quickest line section in S_{pj})

$i := 1$

$\ell := S_{pj}(i)$; $TV_{pj} = \frac{c + f_\ell t_\ell}{f_\ell}$; $f_{pj} = f_\ell$; $S^*_{pj} = \{\ell\}$

3.0 (Try to improve TV_{pj})

$i := i+1$

$\ell := S_{pj}(i)$

if $t_\ell < TV_{pj}$ then

$f_{pj} := f_{pj} + f_\ell$; $TV_{pj} = \frac{TV_{pj} + f_\ell t_\ell}{f_{pj}}$; $S^*_{pj} = S^*_{pj} \cup \{\ell\}$

go to 3

else STOP

In step 1.1 of the assignment algorithm the node labels μ_j, i.e. the expected travel times from the origin, are set to infinity for all nodes except the origin, for which it is initialized to zero. The last transfer nodes on the route between the origin and θ is set to 0 and the set of lines that make minimum the total expected travel time between the origin and θ is arbitrarily defined as $\emptyset$. The set X is used to identify the nodes that are not yet included in the optimal route tree.

In step 1.2, the node nearest to the origin (p) is selected among the nodes not yet included in the optimal route tree. The time considered is μ_p, that is, the total travel time from the origin. In step 3, every node j directly reachable from node p, is labeled by solving the hyperbolic problem that determines the set of line sections which minimize the total expected travel time from p to j. If the new value for μ_j, calculated as the sum of μ_p and TV_{pj} is less than the current value, the label is changed. The calculation of the minimal route tree terminates when all the nodes of the network are included in the tree.

In the second part of the algorithm the demand from the origin node to all the destinations is loaded to the line sections included in the minimal route tree.

5. THE CASE STUDY OF SANTIAGO

The main objective of the tests presented in this section is to compare the results obtained with the strategy choice and the route choice algorithms. To this end our instruments were the EMME/2 package (Babin et al., 1982) where the Spiess' algorithm is imbeded and the CTPM algorithm of TRANSCOM (Chriqui, 1974) implemented in MADITUC (Chapleau et al., 1982) by De Cea and Bunster (1985).

This comparison was performed by using a network that occurs in practice. However, we did not aim to assess the accuracy of the algorithms in reproducing observed flows. We used a node to node O-D matrix obtained from a survey made in 1977 in Santiago (Dictuc, 1978), which obviates any modeling of access to the transit network from centroids.

The network considered is a simplification of the road network of Santiago in 1978. Only those links which are used by transit lines were considered. The network contains 245 nodes, 694 links (unidirectional) and 272 public transport lines having 223 different node sequences. The main characteristic of the Santiago network is the large number of links that carry numerous common lines: 33% of the links have more than 10 common lines and 18% have more than 20 common lines.

Three transportation modes were defined: microbus (171 lines), taxibus (51 lines), both with a constant speed over all the network of 20 km/h and a metro line with a constant speed of 30 km/h. For both assignments we set the parameters of the generalized cost as follows: boarding time := 35 min., wait time factor := 0.5, wait time weight := 1.0, auxiliary transit time weight := 1.0 and boarding time weight := 1.0.

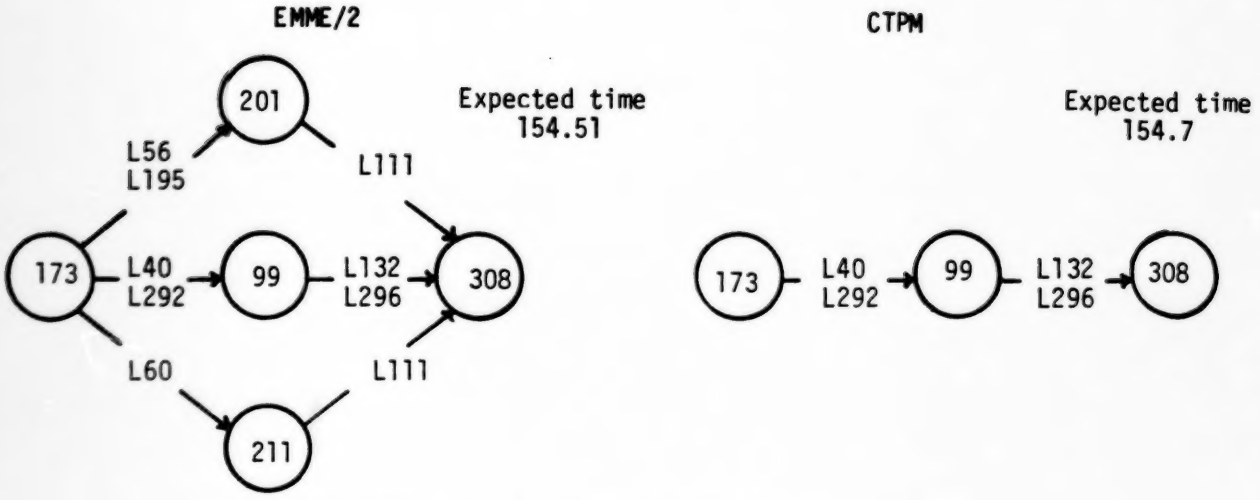
The EMME/2 assignment was performed on a VAX/VMS version 4.3 VAX 11/780 of the Université de Montréal whereas the CTPM test was carried on a VAX/VMS version 4.4 VAX 11/780 at Santiago de Chile (Comision Chilena de

energia nuclear). There seems to be no differences at execution time level between the two versions of the VAX/VMS computers used.

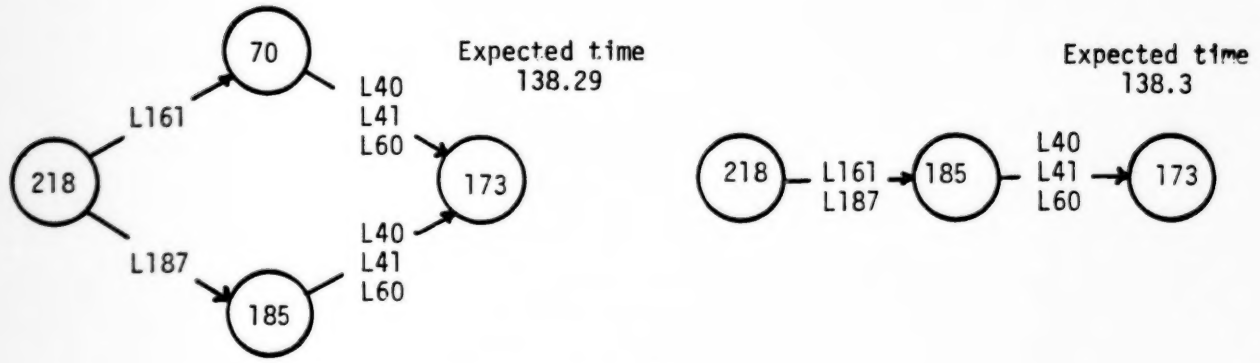
The results of both assignments were compared at three levels of aggregation : disaggregated (itineraries), intermediate (link flows and total line volumes) and global (total generalized times, wait times, number of transfers and execution times).

5.1 Comparison of Itineraries

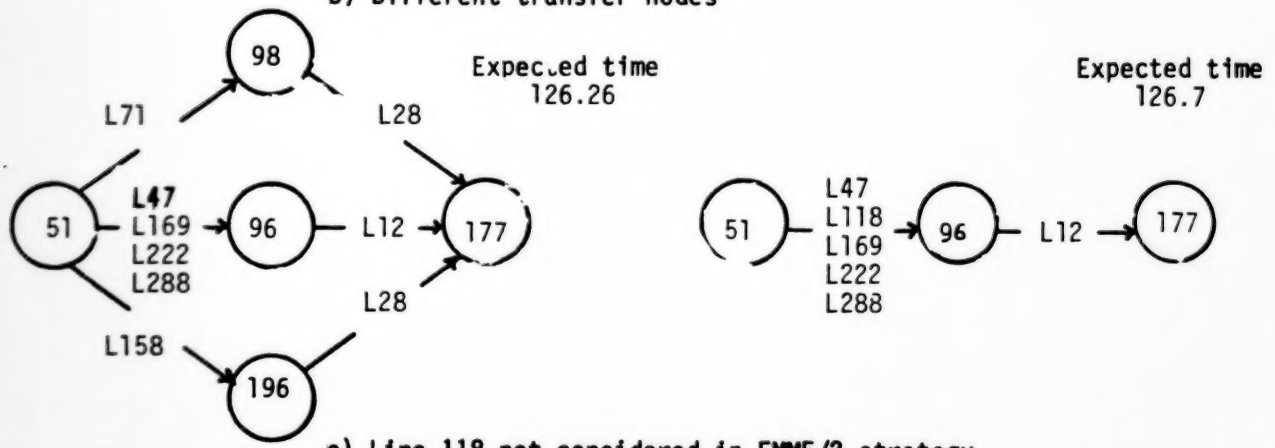
A set of 100 O-D pairs were chosen at random in order to compare the strategies calculated by both algorithms. For those cases where the route does not have transfers (46% of the O-D pairs analyzed), both algorithms produce the same results, as expected. However, when a transfer is needed (54% of the cases) strategies and routes are identical in 61% of the cases, while they differ in the remaining 39%. In order to illustrate the differences, figure 3 shows the strategies chosen by the two algorithms for three O-D pairs. The expected total generalized travel time is indicated at the destination node. Figure 3a shows the case where the Spiess' algorithm detects all possible strategies having the same impedance and CTPM only use a subset of these. In figure 3b, both algorithms use the same transit lines but the transfer nodes are different. The reason for this difference comes from the way the two algorithms perform the labeling procedure : CTPM labels forward, beginning at the origin node whereas the Spiess' algorithm labels backward, starting from the destination node. Finally, we observe one case where a line is used by CTPM (is part of the minimum route) but it is not part of the strategy computed by the Spiess' algorithm (see figure 3c). This is due to the fact that the travel time (without considering the waiting time at node (51)) of the itineraries (51) L118 (96) L112 (177) is greater than the travel times of the itineraries (51) L158 (196) L28 (177) and (51) L71 (98) L28 (177). In fact as the combination of lines 118 and 112 improves the perceived travel time of the route obtained so far, it does not in the case of the strategy.



a) CTPM route is part of EMME/2 strategy



b) Different transfer nodes



c) Line 118 not considered in EMME/2 strategy

Figure 3: Differences Between Strategies and Minimal Routes

5.2 Link volumes and total line boardings

The difference in the strategies used to assign passengers to lines are reflected in the total number of boardings per line and in the total link flows. As EMME/2 considers more possibilities of travel, the distribution of demand among lines will be more even than that obtained with CTPM. In order to illustrate this point, figures 4 and 5 show respectively the actual and relative differences in link flows for downtown Santiago. When the flows produced by EMME/2 are substracted from those produced with CTPM, positive differences are fewer than negative ones. However, we observe in figure 5 high relative differences mainly for low volume links. Table 4 shows a histogram of link volumes vs maximum relative differences in absolute value. Here again, we note major differences only at low volume links.

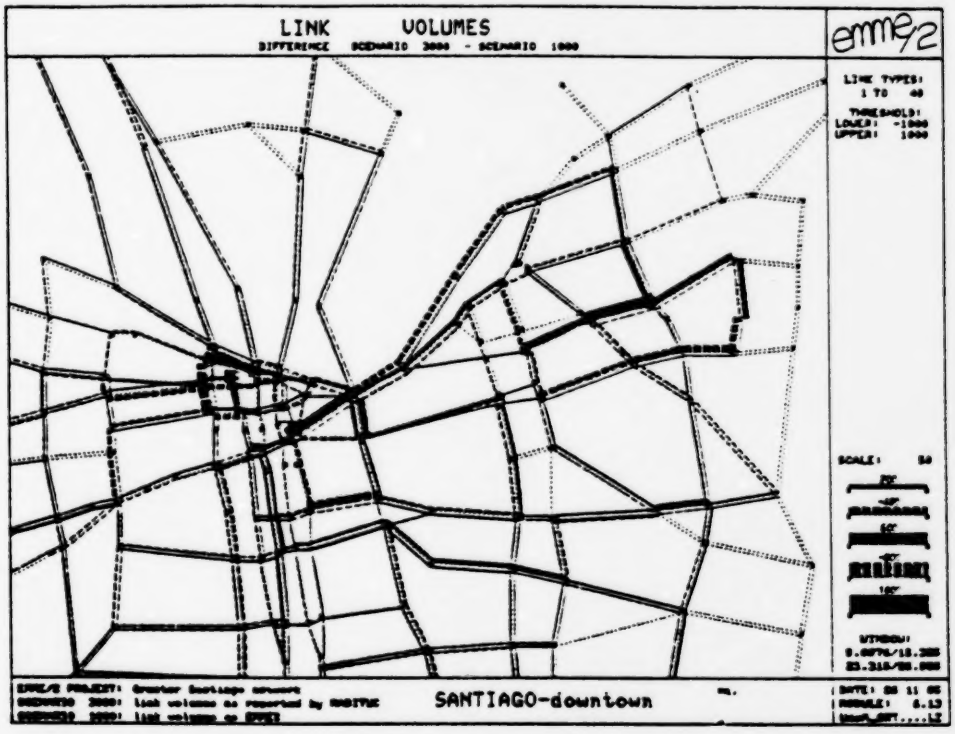


Figure 4: Actual difference in link flows

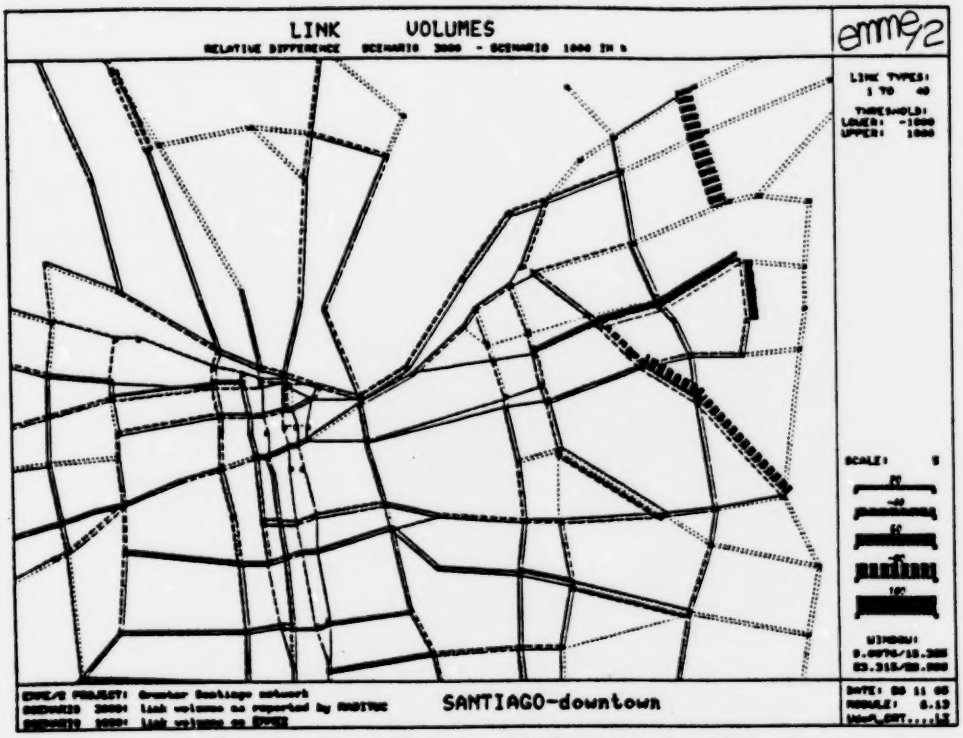


Figure 5: Relative difference in link flows

TABLE 4: Differences in link volumes

Volumes	EMME/2-CTPM in ABS (%)	Frequencies
0	0	63
1 - 999	100.0	131
1000 - 4999	21.2	236
5000 - 9999	8.4	131
10000 - 19999	3.4	82
20000 - 29999	1.6	26
30000 - 39999	1.0	9
40000 - 49999	1.5	14
50000	0.1	2

The differences in total number of boardings are shown in table 5. We note that differences tend to diminish as the total number of boardings

increase. Figure 6 shows a plot of CTPM's boardings vs. EMME/2 boardings. Due to the special line characteristics in Santiago, where there is a high number of common links, the differences in total boardings are not reflected on total link flows.

TABLE 5: Differences in total boardings

Line Volumes	EMME/2-CTPM in ABS (%)	Frequencies
1000	77.2	67
1000 - 2499	38.2	64
2500 - 4999	33.1	54
5000 - 7499	13.7	22
7500 - 9999	19.6	10
10000 - 14999	5.8	3
15000 - 19999	9.6	2
20000	1.3	1

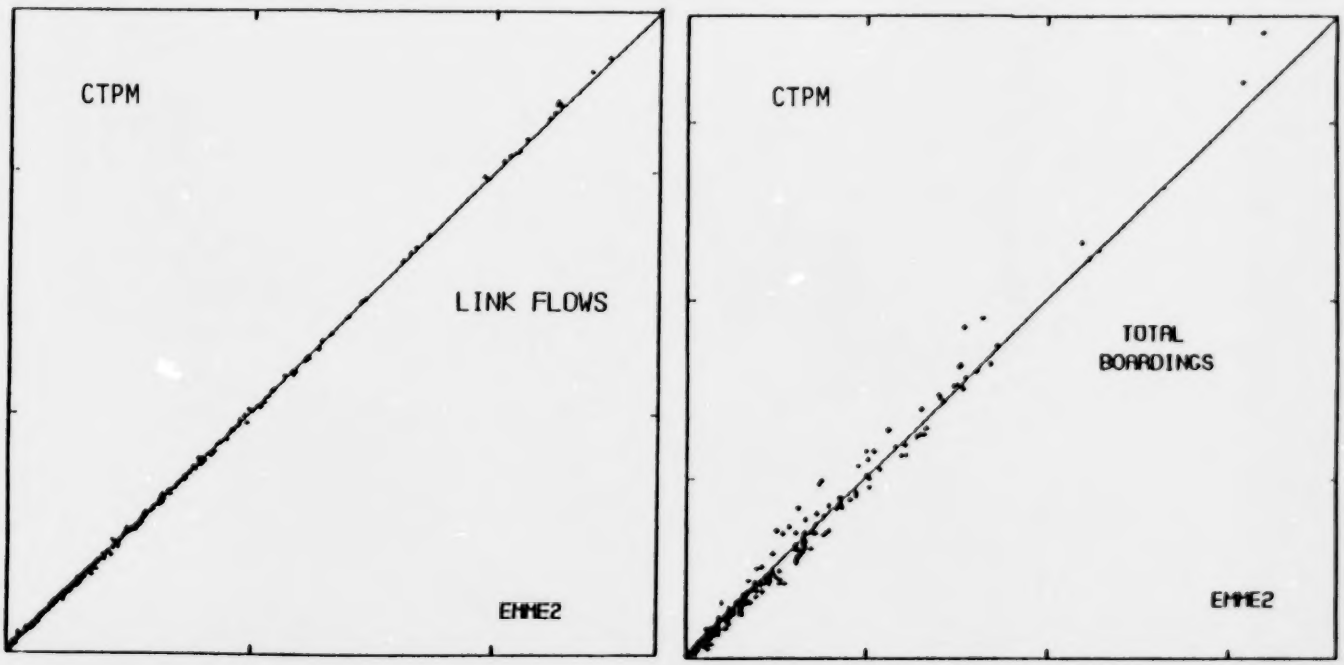


Figure 6: Differences in link flows and total volumes

As we mentioned earlier, the algorithm implemented in EMME/2 does not need to solve explicitly the problem of obtaining the set of attractive lines, whereas the CTPM algorithm must identify from a set of common lines those that minimize the expected total travel time. This stage of the CTPM algorithm is very time consuming and becomes crucial in networks such as Santiago's where more than 60% of the line segments are shared by more than three lines.

In an attempt to reduce this computational effort, De Cea and Bunster (1985) analysed several simplifications of the CTPM algorithm. In these heuristics, the number of transfers and the number of common lines to consider in determining the minimal routes are limited. Below (table 6) we present the results obtained for the case where no limits are imposed on the number of transfers and no common lines are considered in calculating the minimal routes (CTM algorithm) but all those common lines are loaded in the assignment step of the algorithm. This heuristic that does not solve the hyperbolic problem produced the same global statistics as the CTPM algorithm and only 5% of the minimal routes so calculated were different. We observed that the execution time dropped dramatically while still providing a very good solution.

TABLE 6: Global Statistics

	EMME/2	CTPM	CTM
Total demand	504,964	504,964	504,964
Mean travel time	71.34	71.36	71.36
Mean in vehicle time	25.24	25.22	25.22
Mean wait time	2.10	2.15	2.15
Lines/passenger	1.25	1.25	1.25
CPU time (secs.)	263.00	4920.00	835.00[1]

[1] A commercial version inspired in CTM was programmed on FORTRAN 77 for the strategic urban study of Santiago, giving 538 sec. of CPU when tested with the same network of this example.

In table 6, we present global statistics for the three algorithms. The main difference is clearly at execution times, where EMME/2 is eighteen times faster than CTPM and three times faster than CTM.

6. CONCLUSION

We have presented in this paper two transit assignment models in detail: one based on the choice of optimal strategies and one based on the choice of optimal routes. While the computational time required by the optimal strategy algorithm on the network model of the transit services of the city of Santiago is much less than that required by the optimal route algorithm, the differences in the simulated flows are not large.

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